647510 MSC1MatC103x Seat No :________

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination
Oct/Nov. -2019 - [NEW COURSE]
Topology 1(Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q-1) Attempt any seven (each questions carry two marks) : **(7 x 2=14)**

1. Give the Definition of a Topological space.
2. Give the Definition of Sorgen Frey line (Lower limit topology of $\mathbb{R}$).
3. Give an example of neither discrete topology nor indiscrete topology.
4. Give the Definition of Continuous Function in topological space.
5. Give the Definition of Locally Connected Space.
6. Give an example of space which is connected not path connected.
7. Find Closure and Interior point of Rational number and Irrational number.
8. Give the Definition of component.
9. Give the Definition of Basis of topological space.
10. Give an example of neither open nor closed subset of $\mathbb{R}$ with respect to standard topology.

Q-2) Attempt any two : **(2 x 7=14)**

a. State and Prove Hausdorff's Criterion.

b. If $A \subset X$ then $\bar{A} = A \cup A'$.

c. Let X be an infinite set and $\tau = \{G \subset X : G \neq \phi \ and \ X - G \ is \ finite \ \} \cup \{\phi\}$. Check whether τ is topology on X or not ?

Q-3) Attempt the following : **(2 x 7=14)**

1. Let $A = \left\{1, \frac{1}{2}, \frac{1}{3}, . . . \ .\right\} \subset \mathbb{R}$ then prove that $A' = \ \{0\}$, where $\mathbb{R}$ is a topological space with standard topology.

2. Let (X, τ) be a topological space where $\tau = \tau(d)$ for some metric on d on X. Let $E \subset X$ and $x \in X$ then, prove that $x \in \bar{E}$ iff there is a subsequence (x_n) such that $x_n \in E$ for each n and $(x_n) \longrightarrow x$.

OR

Q-3) Attempt the following : **(2 x 7=14)**

1. X locally connected iff each component of each open subset of X is an open subset of X.
2. If X is connected and $f: X \to Y$ is continuous and onto then prove that Y is also connected.

Q-4) Attempt any two : **(2 x 7=14)**

1. If $f: X \to Z$ is continuous and Y a subspace of X then, prove that $f|_Y : Y \to Z$ is also continuous.
2. Consider $\mathbb{R}$ with standard topology. Define $f : \mathbb{R} \to \mathbb{R}$ as $f(x) = x + 1$ Then show that $f : \mathbb{R} \to \mathbb{R}$ is a Homeomorphism.
3. Let X be a non-empty set and $t \in X$ be an element. Let $\tau = \{G \subset X : t \in G\} \cup \{\phi\}$. Prove that τ is a topology on X.

Q-5) Attempt any two : **(2 x 7=14)**

1. If $A, B \subset X$ then $(A \cap B)^{\circ} = A^{\circ} \cap B^{\circ}$.
2. $X \times Y$ is locally connected iff X and Y are locally connected.
3. If A and B are path connected subset of X and $A \cap B \neq \phi$ then $A \cup B$ is path connected.
4. Let (X, d) be a metric space, $x \in X$, $\varepsilon > 0$ and any $y \in B_d(x, \varepsilon)$ then $\exists\ \delta > 0$ such that $B_d(y, \delta) \subset B_d(x, \varepsilon)$.

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